Quantum Computing CO484

Tutorial*

Sheet 5 – Questions

Exercise 1 *What is the matrix for the quantum Fourier transform* $\mathbf{F}$ *when* $n = 1$ *and* $n = 2$*? Show that* $\mathbf{F}$ *is unitary for any* n*.*

Exercise 2 *More on Quantum Fourier Transformation (QFT).*

(i) Work out the matrix for the quantum Fourier transform F *and the network which implements it for* $n = 3$*.*

(ii) Show that the inverse quantum Fourier transform is given by:

$$\mathbf{F}^\dagger : |j\rangle \mapsto \frac{1}{\sqrt{2^n}} \sum_{k=0}^{2^n-1} e^{-2\pi i jk/2^n} |k\rangle . \tag{1}$$

(iii) Work out the circuit for the inverse of the Fourier transform.

Exercise 3 *Quantum Phase Estimation. Let be* $\mathbf{U} : \mathbb{C}^{2^n} \to \mathbb{C}^{2^n}$ *an unitary operator acting on* $\mathbb{C}^{2^n}$ *and let* $|u\rangle \in \mathbb{C}^{2^n}$ *an eigenstate of* $\mathbf{U}$ *and let* $\lambda_{|u\rangle} \in \mathbb{C}$ *its associated eigenvalue:*

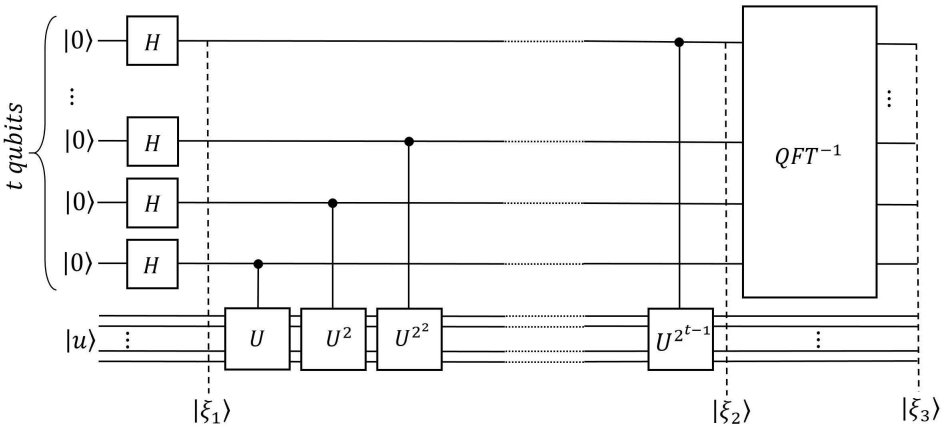
(i) Show that $\exists \phi \in [0, 1[$ *such that* $\lambda_{|u\rangle} = e^{2\pi i \phi}$*.*

(ii) Let t *be a non-zero natural number. Write the form of the states* $|\xi_1\rangle , |\xi_2\rangle$ *as depicted in the quantum circuit below.*

(iii) Suppose that ϕ *is an exact multiple of* $\frac{1}{2^t}$*, write then the form of* $|\xi_3\rangle$*. What does this tell you about* ϕ*?*

Exercise 4 **Prime factorization. Show that any* $n \in \mathbb{N}$ *greater than one can be written as a unique product (up to ordering of the factors) of prime numbers.*

*Partly based on the tutorials by Abbas Edalat and Herbert Wiklicky.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QFT^{-1}
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