Quantum Computing CO484

Tutorial*

Sheet 3 – Questions

Exercise 1 *In the quantum teleportation network of Figure 1, the measurements of the first two qubits by Alice will collapse Bob's qubit as follows:*

$$00 \mapsto |\psi_3(00)\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$01 \mapsto |\psi_3(01)\rangle = \alpha|1\rangle + \beta|0\rangle$$

$$10 \mapsto |\psi_3(10)\rangle = \alpha|0\rangle - \beta|1\rangle$$

$$11 \mapsto |\psi_3(11)\rangle = \alpha|1\rangle - \beta|0\rangle$$

Alice communicates her two bits mn with Bob over a classical channel. Bob will then send his qubit through the circuit $X^n Z^m$ where

$$\mathbf{X} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \mathbf{Z} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Check that the final result $|\psi_4\rangle$ is indeed the state $|\psi_4\rangle = |\psi\rangle = \alpha|0\rangle + \beta|1\rangle$.

Exercise 2 *Let be $f : \{0,1\}^n \to \{0,1\}$ and $\mathbf{U}_f^n : \mathbb{C}^{n+1} \to \mathbb{C}^{n+1}$ with*

$$\mathbf{U}_f^n : |\mathbf{x}, y\rangle \mapsto |\mathbf{x}, y \oplus f(\mathbf{x})\rangle,$$

as depicted in Figure 2. Check that for $n \in \mathbb{N}$ the operator $\mathbf{U}_f^n$ is a unitary transformation.

Exercise 3 *Show that*

$$\mathbf{H}|x\rangle = \frac{1}{\sqrt{2}} \sum_{y \in \{0,1\}} (-1)^{xy} |y\rangle$$

$$\mathbf{H}^{\otimes n}|\mathbf{x}\rangle = \frac{1}{\sqrt{2^n}} \sum_{\mathbf{y} \in \{0,1\}^n} (-1)^{[\mathbf{x},\mathbf{y}]} |\mathbf{y}\rangle$$

where $[\mathbf{x},\mathbf{y}]$ is the bitwise inner product of $\mathbf{x}$ and $\mathbf{y}$ modulo 2.

*Partly based on the tutorials by Abbas Edalat and Herbert Wiklicky.

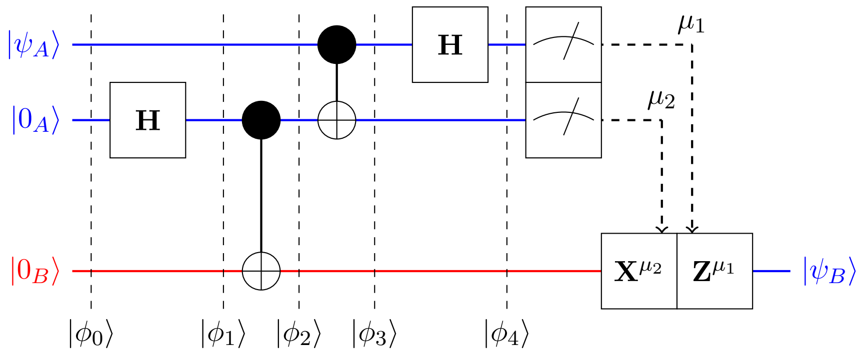

Figure 1: Quantum teleportation

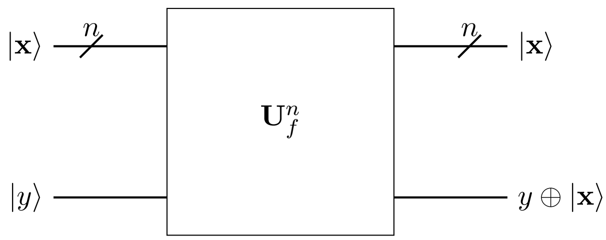

Figure 2: A gate for parallel computation

Exercise 4 **In order to distinguish a function $f : \{0,1\}^n \to \{0,1\}$ from constant to balanced with certainty, one needs at least $2^{n-1}+1$ classical queries. How many classical queries are sufficient for a success probability of $p > \frac{1}{2}$? What does this tell you about the Deutsch-Jozsa problem?*